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14. ABSTRACT

We study stationary solutions of constrained stochastic differential equations driven by fractional Brownian motions. Key motivations for this study originate from the fact that such constrained processes serve as approximation models for a large class of stochastic networks in heavy traffic with long range dependence and self similarity characteristics of data traffic, which are empirically observed in several kinds of local area networks and internet systems. The key mathematical result is a tightness (in time) of the constrained stochastic processes. In a

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Report Title

Constrained Stochastic Differential Equations Driven by Fractional Brownian Motions: Stationarity and parameter estimation problems

ABSTRACT

We study stationary solutions of constrained stochastic differential equations driven by fractional Brownian motions. Key motivations for this study originate from the fact that such constrained processes serve as approximation models for a large class of stochastic networks in heavy traffic with long range dependence and self similarity characteristics of data traffic, which are empirically observed in several kinds of local area networks and internet systems. The key mathematical result is a tightness (in time) of the constrained stochastic processes. In a framework of Stochastic Dynamical Systems (i.e. infinite dimensional state space setting that pertains to noise process with memory), such a tightness result essentially establishes the existence of the stationary solutions. We also address a family of parameter estimation problems for stochastic processes driven by fractional Brownian motions. Parameter estimation problems are usually quite difficult in the physical network models (with or without long memory), whereas the limit stochastic differential models can be much more tractable for statistical analysis.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Conference on Stochastic Processes and Their Applications, Boulder, July 2013.
University of British Columbia, Probability Seminar, April, 2013.
Rutgers University, Probability Seminar, Department of Mathematics, November, 2012.
Invited Lecture, Annual Korean Statistical Meeting, Seoul, Korea, November, 2012.
Northwestern University, Kellogg School of Business, Department of Managerial Economics and Decision Sciences, October, 2012.

Number of Presentations: 5.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

<u>Received</u> <u>Paper</u>

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Received		Paper
08/06/2013	6.00	Yaozhong Hu, Chihoon Lee, Myung Hee Lee, Jian Song. Parameter estimation for reflected Ornstein-Uhlenbeckprocesses with discrete observations, Statistical Inference for Stochastic Processes (03 2013)
08/06/2013	7.00	Libo Sun, Chihoon Lee, Jennifer Hoeting. Penalized importance sampling for parameter estimation in stochastic differential equations, Biometrics (05 2013)

TOTAL: **2**

Number of Manuscripts:

Books

Received	Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Early Career Statistician Award, Korean Statistical Society, November 2012

Graduate Students

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Chihoon Lee	1.00	
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
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Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

Final Report[1]

Starting Date: 09/10/2012, End Date: 06/09/2013 (ARO, STIR program)

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1 Statement of the Problem and Summary of the Results

In the first part of this project, we study stationary solutions of constrained stochastic differential equations (SDEs) driven by fractional Brownian motions (FBMs). Motivations for this study originate from the fact that such constrained processes serve as an approximation for a large class of high-speed communications stochastic networks in heavy traffic, such as local area networks and internet systems. To study ergodic properties of a dynamical system with memory, we investigate the ideas based on Markovian random dynamical systems and establish a key tightness (in time) result of the constrained stochastic processes. In a framework of Stochastic Dynamical Systems (i.e. infinite dimensional state space setting that pertains to noise process with memory), such a tightness result essentially establishes the existence of the stationary solutions.

In the second part of this project, we address several parameter estimation problems for a class of one-dimensional reflected SDEs driven by FBMs. We note that parameter estimation problems are usually quite challenging in the physical network models (with or without long memory), whereas the limit SDE models can be much more tractable for statistical analysis. We derive some explicit form of maximum likelihood and method of moment estimators for the drift parameters, then obtain their desirable statistical properties, which are based on continuous time and discrete time observations, respectively.

In the third part of this project, we justify the use of constrained SDEs as approximation models for a very general class of stochastic networks in heavy traffic with non-Markovian state dependence; long range dependence can be considered as a special case of such broader assumptions. We establish heavy traffic limit theorems for queue-length processes in critically loaded single class queueing networks with state dependent arrival and service rates. The limit stochastic process is a continuous-path reflected process on the nonnegative orthant. The constrained stochastic differential equation driven by FBM is a special example of such limit stochastic processes. We also give an application to the classical generalized Jackson networks with state-dependent rates.

Overall, the conducted research would advance the existing knowledge in the steady-state behavior and control issues of stochastic networks operating under heavy traffic regime. The work is expected to have impacts beyond its theoretical contributions in applied probability, with advances likely to be influential in operations research and defense-related strategic

[1]Research supported by US Army Research Office W911NF-12-1-0494.

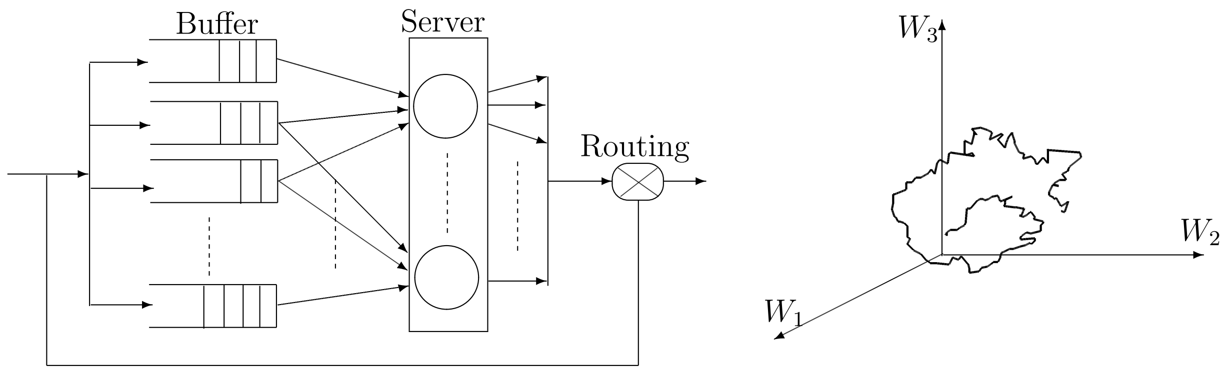

Figure 1: *An illustration of a typical stochastic network (at left) and its approximating limit model (at right). The coordinate W_i's represent the weak limit of the suitably scaled queue-length (or workload) processes for service stations of the underlying network system. The approximations give parsimonious mathematical models that are much more tractable for statistical inference as well as long time asymptotic analysis.*

engineering. Design of efficient communication networks increasingly relies on theoretical approximations and statistical parameter estimations such as those to be developed here. While the research is motivated by stochastic network data, the methodologies could be utilized in applications well beyond the network setting and likely to be extended and adapted by researchers from many other areas of science and engineering where long range dependence and self similarity are key features.

2 Stationary Solutions to Constrained SDEs Driven by FBMs

The SDE model under consideration is given by

$$W(t) = W(0) + \int_0^t b(W(s))ds + B_H(t) + \eta(t) \in \mathcal{X}, \quad t \geq 0,$$

where B_H is a standard fractional Brownian motion (with appropriate dimension) and $\mathcal{X}$ is a given (polyhedral) domain. Roughly speaking, the nonnegative process $\eta(\cdot)$ describes the *minimal*, continuous correctional term with finite variation, and increasing only when $\{W(t)\}$ is on the boundary $\partial\mathcal{X}$. In other words, the processes $\{W(t), \eta(t)\}$ solve the Skorokhod problem

$$W(t) = \Gamma\left(W(0) + \int_0^{\cdot} b(W(s))ds + B_H(\cdot)\right)(t) \in \mathcal{X}, \quad t \geq 0,$$

where $\Gamma(\cdot)$ is the Skorohod (reflection) map. Based upon such a representation, we say that $\{W(t)\}_{t\geq 0}$ is a *stationary* solution to the above equation if

$$\left(W(t) - W(0) - \int_0^t b(W(s))ds - \eta(t)\right)_{t\geq 0} \stackrel{\mathrm{d}}{=} (B_H(t))_{t\geq 0},$$

where the equality is the equality of all finite dimensional distributions, and their joint probability distribution does not change when shifted in time. With this setup, our main goal is to establish the existence and uniqueness of the stationary solution under the suitable stability condition on the drift term $b(\cdot)$. The main idea is based on studying sequences of empirical measure of Euler schemes described below.

Fix a constant step size $\gamma > 0$, then denote $(\bar{W}_{k\gamma})_{k\geq 0}$ the Euler scheme defined by

$$\bar{W}_0 = x \in \mathcal{X}, \quad \bar{W}_{(k+1)\gamma} = \bar{\Gamma}\left(\bar{W}_{k\gamma}, \gamma b(\bar{W}_{k\gamma}) + \Delta_{k+1}\right),$$

where

$$\Delta_{k+1} = B_H((k+1)\gamma) - B_H(k\gamma)$$

is a fractional Gaussian noise and $\bar{\Gamma}(\bar{w}, \cdot)$ is the Skorokhod map Γ evaluated at time 1 with initial condition $\bar{w} \in \mathcal{X}$. Then, a sequence of random probability measures $(\bar{\nu}^{(n,\gamma)}(\omega, d\alpha))_{n\geq 1}$ is defined on the Skorokhod space $D(I\!R_+, \mathcal{X})$ by

$$\bar{\nu}^{(n,\gamma)}(\omega, d\alpha) = \frac{1}{n}\sum_{k=1}^{n} \delta_{\bar{W}_{(k-1)\gamma}(\omega)}(d\alpha),$$

where δ denotes the Dirac measure. This random measure constitutes our basic sequence of approximations for the stationary solution to the constrained SDE models. Under suitable stability conditions on the drift vector field $b(\cdot)$ and regularity of the Skorokhod map, we are to establish that $\bar{\nu}^{(n,\gamma)}(\omega, d\alpha)$ for large n and small γ are natural approximations of the stationary solutions.

A fundamental step is the *tightness* of $(\bar{\nu}^{(n,\gamma)}(\omega, d\alpha))_{n\geq 1}$. For the tightness result, we establish that

$$\sup_{n\geq 1} \frac{1}{n}\sum_{k=1}^{n} |\bar{W}_{(k-1)\gamma}|^2 < +\infty \quad \text{a.s.}$$

for sufficiently small step size $\gamma \in (0, \infty)$. The stationary and ergodic properties of the fractional Gaussian noise sequence $(\Delta_k)_{k\geq 1}$ along with suitable stability properties of the Skorokhod map play an important role in the proof of the above estimate.

Next, we identify that every weak limit of $(\bar{\nu}^{(n,\gamma)}(\omega, d\alpha))_{n\geq 1}$, denoted by $\bar{\nu}^{(\infty,\gamma)}(\omega, d\alpha)$, is indeed *a.s.* a stationary Euler scheme with step γ of SDE model. More precisely, we denote by $\{\bar{W}_t^{(\gamma)}\}_{t\geq 0}$ the stepwise constant continuous-time Euler scheme defined by $\bar{W}_t^{(\gamma)} = \bar{W}_{k\gamma}$ for all $t \in [k\gamma, (k+1)\gamma)$. Then, we show that a.s. $\bar{\nu}^{(\infty,\gamma)}(\omega, d\alpha)$ is the distribution of an RCLL process $\mathbb{W}^{(\infty,\gamma)} = (\mathbb{W}_t^{(\infty,\gamma)})_{t\geq 0}$, which is a *discretely* stationary solution.

Then the last important step is to prove that, a.s., $(\bar{\nu}^{(\infty,\gamma)}(\omega, d\alpha))_{\gamma>0}$ is tight for the weak topology induced by the topology of the uniform convergence on $D(I\!R_+, \mathcal{X})$ and that its weak limits when $\gamma \to 0$ are stationary solutions to our reflected SDE model. The main difficulty arises when we establish the tightness of $(\bar{\nu}^{(\infty,\gamma)}(\omega, d\alpha))_{\gamma}$. For this step, we focus on the particular case $b(x) = -x$ (i.e., $\{W_t\}_{t\geq 0}$ is a reflected fractional Ornstein-Uhlenbeck process) where some explicit computations lead to a control of $(\bar{\nu}^{(\infty,\gamma)}(\omega, d\alpha))_{\gamma}$ for fixed

$\gamma > 0$. Then we show this control can be extended to constrained SDE model whose drift term satisfies more general stability conditions. A joint paper [6] with Jian Song (University of Hong Kong) based on this topic is currently under preparation.

3 Parameter Estimation for Constrained SDEs Driven by FBMs

3.1 Sequential parameter estimation for reflected fractional Ornstein-Uhlenbeck processes

To illustrate our problems, consider the following one-dimensional reflected fractional Ornstein-Uhlenbeck (RFOU) process $\{W_t\}_{t\geq 0}$ on the positive real line:

$$\left.\begin{aligned} dW_t = -\gamma W_t dt + \sigma dB_t^H + dL_t, \\ W_t \geq 0 \quad \text{for all } t \geq 0, \end{aligned}\right\}$$

where $\gamma \in (0, \infty)$, $\sigma \in (0, \infty)$ and $B^H = (B_t^H)_{t\geq 0}$ is a one-dimensional fractional Brownian motion with the Hurst parameter $H \in [1/2, 1)$. For example, the above reflected fractional Ornstein-Uhlenbeck process arises as the key approximating process for queueing systems with reneging or balking customers (with long range dependent arrival process). The drift parameter γ carries the physical meaning of customers' reneging (or, balking) rate from the system. In a recent work of PI [5], the goal is

> to estimate the unknown drift parameter $\gamma \in (0, \infty)$ based on observations of the state process $\{W_t\}_{t\geq 0}$.

When the driving noise is a standard Brownian motion (i.e., $H = 1/2$), the sequential estimation plan for the reflected Ornstein-Uhlenbeck process has been studied in the PI's recent paper [3]. More precisely, the process $\{W_t\}$ is assumed to be observed until the observed Fisher information of the process exceeds a predetermined level of precision h, i.e., we observe $\{W_t\}$ over the random time interval $[0, \tau(h)]$ where the stopping time $\tau(h)$ is defined as

$$\tau(h) := \inf\left\{t \geq 0 : \int_0^t W_s^2 ds \geq h\right\}, \quad 0 < h < \infty, \tag{1}$$

and the $\mathcal{F}^W_{\tau(h)}$-measurable function $\widehat{\gamma}_{\tau(h)}$

$$\widehat{\gamma}_{\tau(h)} := \frac{1}{h}\left[bL_{\tau(h)} - \int_0^{\tau(h)} W_s dW_s\right], \tag{2}$$

is a sequential estimator of the parameter γ.

Recenlty, we obtain the sequential estimation plan for the reflected fractional Ornstein-Uhlenbeck process ($H \in (0, 1)$) and verify that the proposed plan is significantly helpful

both in asymptotic and non-asymptotic short time observation; see [5]. The main technical approach is based on the so-called Molchan martingale, that is, letting

$$M_t^H := \int_0^t c_H s^{\frac{1}{2}-H}(t-s)^{\frac{1}{2}-H} I_{\{0<s<t\}} dB_s^H$$

with a suitable constant $c_H > 0$, $\{M_t^H\}$ is a Gaussian martingale with square bracket $\langle M\rangle_t^H = t^{2-2H}$. The stopping time defined by, for any $h > 0$,

$$\tau^H(h) := \inf\{t > 0 : \int_0^t \chi_s^2 d\langle M\rangle_s^H \geq h\} \tag{3}$$

plays the similar role as in the standard Brownian motion case (1). Here, the process χ is a suitably constructed functional of the observable process W so that one can take advantage of the square integrable martingale property of the process $\{\int_0^t \chi_s dM_s^H : t > 0\}$.

With the stopping time (3), we construct a sequential estimator analogous to (2) and prove that the sequential estimation plan $(\tau^H(h), \widehat{\gamma}^H_{\tau^H(h)})$ has the following desirable properties:

> (a) it is unbiased; (b) the estimation plan is closed, i.e., the time of the observation $\tau^H(h)$ is finite with probability 1; (c) its mean squared error is a constant that does not depend on the parameter γ to be estimated.

Such results would be of ample use in applications to several areas such as engineering, financial and biological modeling where unknown parameter estimation is based on relatively shorter time observation, which commonly arises in practical situations.

A paper [5] based on this project is submitted and currently under review for *Stochastic Processes and Their Applications*.

3.2 Parameter estimation for ROU based on discrete observations

For a fixed time interval $h > 0$, assume that the discrete time process $\{W_{kh} : 1 \leq k \leq n\}$ are observable to the system manager. In the PI's recent work [2], the following simple formula for the stationary moment of the reflected Ornstein-Uhlenbeck process ($H = 1/2$) is obtained: For all $\alpha > -1$, $\alpha \neq 0$,

$$\mathbb{E}|W(\infty)|^\alpha = \Gamma\left(\frac{\alpha+1}{2}\right)\frac{\sigma^\alpha}{\sqrt{\pi}}\gamma^{-\alpha/2},$$

where $\Gamma(\cdot)$ denotes the Gamma function. Hence, a natural method of moment estimator of γ is given by

$$\hat{\gamma}_{\alpha,n} := \frac{(\Gamma((\alpha+1)/2))^{2/\alpha}\,\sigma^2}{\pi^{1/\alpha}}\left(\frac{1}{\frac{1}{n}\sum_{k=1}^n |W(kh)|^\alpha}\right)^{2/\alpha}$$

for an arbitrary $h > 0$ and any $\alpha > -1$, $\alpha \neq 0$. Strong consistency and asymptotic normality of the method of moment estimator are established in [2].

The two main ingredients in the asymptotic analysis are ergodic properties of the constrained stochastic process $\{W(t)\}_{t\geq 0}$ and the method of moments estimator constructed from the stationary distribution of $W(\infty)$.

A paper [2] based on this project is submitted and currently under review for *Statistical Inference for Stochastic Processes*.

3.3 Drift parameter estimation for reflected FBM process based on the local time process

In the PI's recent work [1], we consider the one-dimensional reflected FBM process model with constant drift parameter b. Suppose the observation process is given by a local time process $\{L(t)\}_{t\geq 0}$ (instead of the state process $\{W(t)\}_{t\geq 0}$), which can be interpreted as the total time the queue has been empty up to time t. We obtain an estimator for the drift parameter b based on the observed cumulative local time process L, and establish its strong consistency and asymptotic normality. More precisely, for $0 < H < 1$, we define an estimator $\hat{b}_T = -\frac{1}{T}L(T)$. Then, the strong consistency follows:

$$\hat{b}_T \to b \ \text{ as } \ T \to \infty \ \text{ a.s.}$$

Also, the asymptotic normality result for the estimator $\hat{b}_T$ can be shown, for $1/2 \leq H < 1$,

$$T^{1-H}(\hat{b}_T - b)/\sigma \Rightarrow N(0,1) \ \text{ as } \ T \to \infty.$$

The proof is based on explicit representations of the state and local time processes and the asymptotic results on the tail distribution of the transient queue driven by FBM.

A paper [1] based on this project has recenlty published in *Journal of Applied Probability*. As a separate (not in the network context) but remotely related parameter estimation project described in [7], we have proposed a penalized importance sampling estimator to obtain computationally efficient numerical schemes in estimating parameters for a widely applicable class of multidimensional SDE models.

4 Non-Markovian state-dependent networks in critical loading

Queueing systems with arrival and (or) service rates depending on the system's state arise in various application areas which include manufacturing, storage, service engineering, and communication and computer networks. Longer queues may lead to customers being discouraged to join the queue, or to faster processing, e.g., when human servers are involved. State-dependent features are present in congestion control protocols in communication networks, such as TCP. In a joint paper with Puhalskii [4], we consider an open network of single server queues and establish limit theorems for non-Markovian state-dependent networks in critical loading.

A distinguishing feature of the model is *non-Markovian state dependence*. More specifically, the number of customers at station i, where $i = 1, 2, \dots, K$, is governed by the following equations

$$
\begin{aligned}
Q_i(t) =& Q_i(0) + A_i(t) + B_i(t) - D_i(t), \\
A_i(t) =& N_i^A \left(\int_0^t \lambda_i(Q(s)) ds \right), \\
B_i(t) =& \sum_{j=1}^K \Phi_{ji}\big(D_j(t)\big), \\
D_i(t) =& N_i^D \left(\int_0^t \mu_i(Q(s)) 1_{\{Q_i(s)>0\}} ds \right),
\end{aligned}
\tag{4}
$$

where $Q(s) = (Q_1(s), \dots, Q_K(s))$ denotes the vector of the queue lengths at the stations at time s.

The quantities $N_i^A(t)$ and $N_i^D(t)$ represent the number of exogeneous arrivals and the maximal number of customers that can be served, respectively, at station i by time t under "nominal" conditions, $\Phi_{ji}(m)$ denotes the number of customers routed from station j to station i out of the first m customers served at station j, and $\lambda_i(Q(t))$ and $\mu_i(Q(t))$ represent instantaneous exogenous arrival and service rates, respectively, for station i at time t given the queue length vector $Q(t)$. Generalised Jackson networks is a special case of (4) where the N_i^A and N_i^D are renewal processes, the Φ_{ij} are Bernoulli processes, and $\lambda_i(\cdot) = \mu_i(\cdot) = 1$.

We obtain limit theorems in critical loading for the queue length processes akin to classical diffusion approximation results available for generalised Jackson networks. The limit stochastic process is a continuous-path reflected process (generally, non-Markovian) on the nonnegative orthant. The constrained stochastic differential equation driven by FBM is a special example of such limit stochastic processes. We give an application to generalised Jackson networks with state-dependent rates.

The main contribution of our work [4] is incorporating general arrival and service processes. This is achieved by applying an approach different from the ones used by other related works. In our approach we, in a certain sense, return to the basics and employ ideas which have proved their worth in the set-up of generalised Jackson networks. We show that continuity considerations may produce stronger conclusions at less complexity. Our main result states that if the network primitives satisfy certain limit theorems with continuous-path limits, then the multidimensional queue-length processes, when suitably scaled and normalised, converge to a reflected continuous-path process on the nonnegative orthant.

A paper [4] based on this project is submitted and currently under review for *Stochastic Models*.

References

[1] Yaozhong Hu and Chihoon Lee. Drift parameter estimation for reflected fractioinal Brownian motions based on local time processes. *Journal of Applied Probability*, 50(2):592–597,

2013.

[2] Yaozhong Hu, Chihoon Lee, Myung Hee Lee, and Jian Song. Parameter estimation for reflected Ornstein-Uhlenbeck processes with discrete observations. Under review in *Statistical Inference for Stochastic Processes*, 2013.

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[4] Chihoon Lee and Anatolii A. Puhalskii. Non-Markovian state-dependent networks in critical loading. Under review in *Stochastic Models*, 2013.

[5] Chihoon Lee and Jian Song. On drift parameter estimation for reflected fractional Ornstein-Uhlenbeck processes. Under review in *Stochastic Processes and Their Applications*, 2013.

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[7] Libo Sun, Chihoon Lee, and Jennifer Hoeting. Penalized importance sampling for parameter estimation in stochastic differential equations. Under review in *Biometrics*, 2013.